

FACTS ABOUT EXPONENTIALS AND LOGARITHMS

I. Definitions of Exponents: We assume m and n are *counting numbers*.

	<i>Definition:</i>	<i>Example:</i>
1. Counting number exponents:	$x^1 = x$ if $n > 0$ then $x^{n+1} = x \cdot x^n$	$5^1 = 5$ $x^5 = x \cdot x^4 = x \cdot x \cdot x \cdot x \cdot x$
2. Zero exponent:	$x^0 = 1$	$5^0 = 1$
3. Negative exponent:	$x^{-n} = \frac{1}{x^n}$	$x^{-2} = \frac{1}{x^2}$
4. Fractional exponent:	$x^{\frac{1}{n}} = \sqrt[n]{x}$ $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$ $x^{-\frac{m}{n}} = \frac{1}{(\sqrt[n]{x})^m}$	$x^{\frac{1}{2}} = \sqrt{x}$ $x^{\frac{2}{3}} = (\sqrt[3]{x})^2$ $x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$

II. Definitions of Logarithm: We assume the base a is a *positive number* and not 1. These are equivalent:

- $\log_a x$ is the exponent of a that gives x .
($\log_a x$ es la potencia a la que debe elevarse a para obtener x .)
- $n = \log_a x$ means $a^n = x$
- $a^{\log_a x} = x$
- $\log_a a^n = n$

III. Special logarithms:

- $\log x = \log_{10} x$ So: $\boxed{10^{\log x} = x}$ and $\boxed{\log 10^x = x}$
- $\ln x = \log_e x$ where $e = 2.71828\dots$ So: $\boxed{e^{\ln x} = x}$ and $\boxed{\ln e^x = x}$

IV. Change of Base for Logarithms:

$$\log_b x = \frac{\log x}{\log b} \quad (\text{On the right side, "log" can be to any base.})$$

V. The THREE GOLDEN RULES:

Exponents:

$$1. a^m \cdot a^n = a^{m+n}$$

$$2. \frac{a^m}{a^n} = a^{m-n}$$

$$3. (a^m)^n = a^{nm}$$

Logs:

$$1. \log_a(xy) = \log_a x + \log_a y$$

$$2. \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$3. \log_a(x^n) = n(\log_a x)$$

VI. Functions and Inverses:

Inverses:

Examples:

Function name	Inverse name	Function	Inverse	Function	Inverse
add	subtract	$y = x + a$	$y = x - a$	$y = x + 3$	$y = x - 3$
multiply	divide	$y = ax$	$y = \frac{x}{a}$	$y = 3x$	$y = \frac{x}{3}$
n th power	n th root	$y = x^n$	$y = \sqrt[n]{x}$	$y = x^2$	$y = \sqrt{x}$
exponential	logarithm	$y = a^x$	$y = \log_a x$	$y = 2^x$	$y = \log_2 x$

VII. Forbidden Errors:

$$1. x^a + x^b \text{ is NOT equal to } x^a \cdot x^b \text{ or } x^{a+b}$$

$$2. x^a - x^b \text{ is NOT equal to } \frac{x^a}{x^b} \text{ or } x^{a-b}$$

$$3. x^a - x^b \text{ is NOT equal to } x^{\frac{a}{b}}$$

$$4. x^{a^n} \text{ is NOT equal to } x^{na} \text{ or } (x^a)^n$$

$$5. x^1 \text{ is NOT equal to } 0 \text{ or } 1$$

$$\log(x+y) \text{ is NOT equal to } \log x + \log y \text{ or } \log xy$$

$$\log(x-y) \text{ is NOT equal to } \log\left(\frac{x}{y}\right) \text{ or } \frac{\log x}{\log y}$$

$$\frac{\log x}{\log y} \text{ is NOT equal to } \log x - \log y$$

$$(\log x)^n \text{ is NOT equal to } n(\log x)$$

$$\log 0 \text{ is NOT equal to } 1 \text{ or } 0$$